

# Two-Photon Decay of Neutral Pion from Lattice QCD

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BNL, New York, 2012/05/14

# Motivation

- PrimEx@JLab:  $\Gamma_{\pi^0\gamma\gamma} = 7.82(22)$  eV [PrimEx, PRL106, 2011]
- Precision: 2.8% → 1.4% (projected goal)
- Benchmark test in the chiral anomaly sector of QCD
- $\Gamma_{\pi^0\gamma\gamma} = (\pi/4)\alpha_e^2 m_\pi^3 \mathcal{F}_{\pi^0\gamma\gamma}^2(m_\pi^2, 0, 0)$ 
  - ▶  $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$ :  $\pi^0 \rightarrow \gamma^* \gamma^*$  transition form factor
  - ▶  $m_\pi$ :  $\pi^0$  mass;  $p_1, p_2$ : photon 4-momentum

## Lattice setup

- overlap fermion: exact chiral symmetry  $\Rightarrow$  test chiral anomaly
- all-to-all propagators  $\Rightarrow C(t_1, t_2, t_3)$ 
  - ▶ low mode: low-lying eigenvalues and eigenvectors
  - ▶ high mode: stochastic propagators
  - ▶ calculate correlator at any time slice of  $t_1, t_2, t_3$
  - ▶ disconnected diagram

## Ensemble information

- four  $m_{u,d}$ :  $m_\pi = 540 \rightarrow 290$  MeV  $\Rightarrow$  chiral extrapolation
- $m_s$  fixed to be close to its physical value  $\Rightarrow$  dynamical  $s$ -quark effects
- $L/a = 16$  and  $24$   $\Rightarrow$  finite-size effects
- $Q = 0$  and  $1$   $\Rightarrow$  fixing-topology effects
- $a = 0.11$  fm  $\Rightarrow$  study possible lattice artifacts

# Theoretical setup

- starting point: S-matrix

$$\langle \gamma(p_1, \lambda_1) \gamma(p_2, \lambda_2) | \pi^0(q) \rangle$$

- transition form factor is defined by matrix element

$$\underbrace{\int d^4x e^{ip_1 x} \langle \Omega | T\{J_\mu(x) J_\nu(0)\} | \pi^0(q) \rangle}_{\mathcal{M}_{\mu\nu}} = \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta F_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$$

- ▶  $\epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta$ : induced by the negative parity of the  $\pi^0$
- ▶ chiral limit, photon on-shell, ABJ anomaly:

$$\mathcal{F}_{\pi^0\gamma\gamma}(0, 0, 0) = \mathcal{F}_{\pi^0\gamma\gamma}^{\text{ABJ}} = \frac{1}{4\pi^2 F_\pi}$$

# Method

- matrix element in the Minkowski space-time

$$\mathcal{M}_{\mu\nu} = \int d^4x e^{ip_1 x} \langle \Omega | T\{J_\mu(x) J_\nu(0)\} | \pi^0(q) \rangle$$

- analytical continuation from Minkowski to Euclidean space-time  
[Ji, Jung, 2001; Dudek, Edwards, 2006]

$$\mathcal{M}_{\mu\nu} = \int dt e^{\omega t} \int d^3\vec{x} e^{-i\vec{p}_1 \cdot \vec{x}} \langle \Omega | T\{J_\mu(x) J_\nu(0)\} | \pi^0(q) \rangle$$

- pion is on-shell:  $q = (E_\pi, \vec{q}) \Rightarrow q^2 = m_\pi^2$
- photon 4-momentum:  $p_1 = (\omega, \vec{p}_1)$  and  $p_2 = (E_\pi - \omega, \vec{q} - \vec{p}_1)$
- requirement:  $p_{1,2}^2 < m_\rho^2$  or  $E_{\pi\pi}^2$  (hadron production threshold)

# Amplitude $A_\pi(\tau)$

- observable:

$$\mathcal{M}_{\mu\nu} = \int dt e^{\omega t} \int d^3 \vec{x} e^{-i \vec{p}_1 \cdot \vec{x}} \langle \Omega | T\{ J_\mu(x) J_\nu(0) \} | \pi^0(q) \rangle$$

- correlation function:

$$C_{\mu\nu} = \langle \Omega | J_\mu(\vec{p}_1, t_1) J_\nu(\vec{p}_2, t_2) \pi^0(-\vec{q}, t_\pi) | \Omega \rangle$$

- set  $\tau = t_1 - t_2$  and  $t = \min\{t_1, t_2\}$

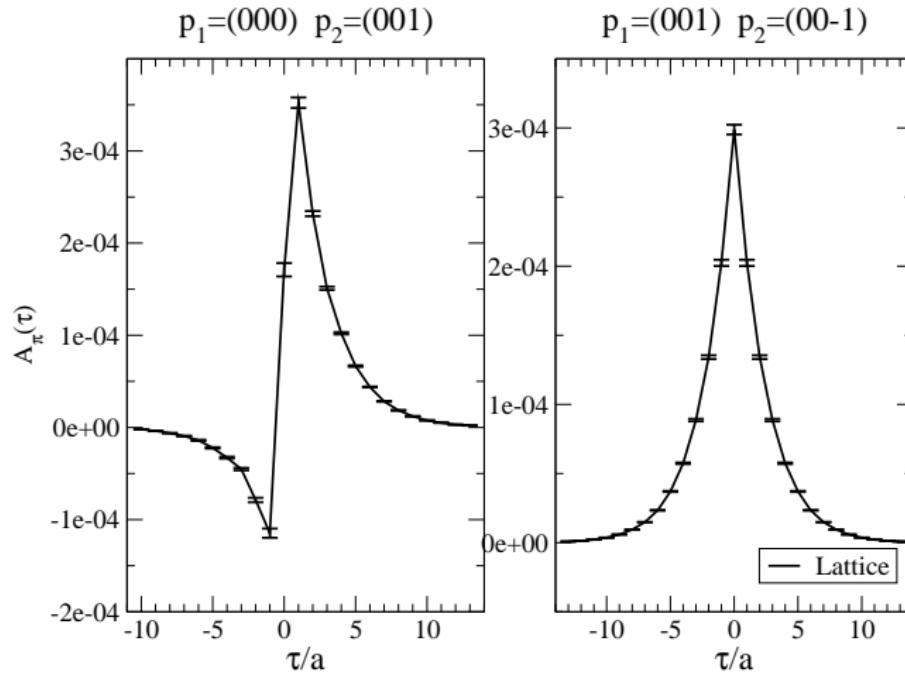
$$A_\pi(\tau) \equiv \lim_{t-t_\pi \rightarrow \infty} C_{\mu\nu}(t_1, t_2, t_\pi) / e^{-E_\pi(t-t_\pi)}$$

- matrix element  $\mathcal{M}_{\mu\nu}$  is given by

$$\mathcal{M}_{\mu\nu} = \frac{2E_\pi}{\phi_\pi} \left( \int_0^\infty d\tau e^{\omega\tau} A_\pi(\tau) + \int_{-\infty}^0 d\tau e^{(\omega-E_{\pi,\vec{q}})\tau} A_\pi(\tau) \right)$$

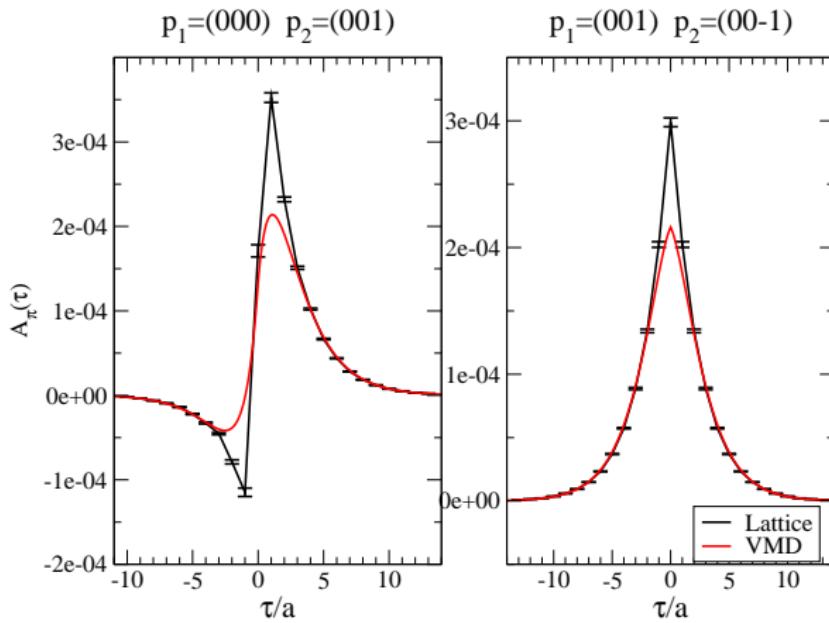
# Distribution of $A_\pi(\tau)$

- $\langle J_\mu(\vec{p}_1, t_1) J_\nu(\vec{p}_2, t_2) \pi^0(\vec{q}, t_\pi) \rangle$ :  $\mu = 1, \nu = 2$



# Time dependence of $A_{\pi,1,2}$

- VMD model:  $\mathcal{F}_{\pi^0\gamma\gamma}^{\text{VMD}}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2)$
- $G_V(p^2) = M_V^2 / (M_V^2 - p^2)$  is the vector meson propagator,  $M_V = m_\rho$



## Fit ansatz

- lowest vector meson effects should be accounted for first
- corrected by including excited-state effects

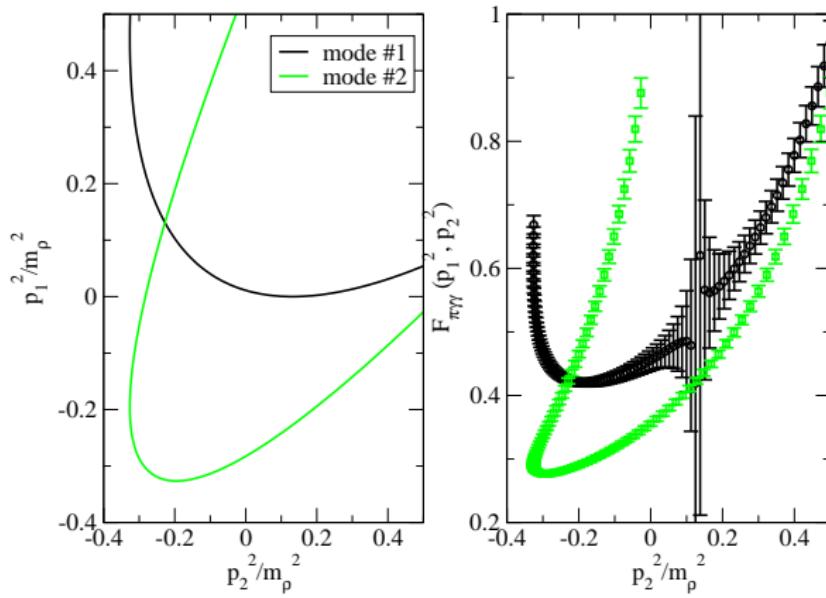
$$\begin{aligned}\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) &= c_V G_V(p_1^2) G_V(p_2^2) \\ &+ \sum_{V'} c_{V'} (G_V(p_1^2) G_{V'}(p_2^2) + G_{V'}(p_1^2) G_V(p_2^2)) \\ &+ \sum_{V', V''} c_{V'V''} G_{V'}(p_1^2) G_{V''}(p_2^2)\end{aligned}$$

- replace  $G_{V'}(p^2)$  by a basis of polynomial function

$$\begin{aligned}\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) &= c_V G_V(p_1^2) G_V(p_2^2) \\ &+ \sum_m c_m ((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)) \\ &+ \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n\end{aligned}$$

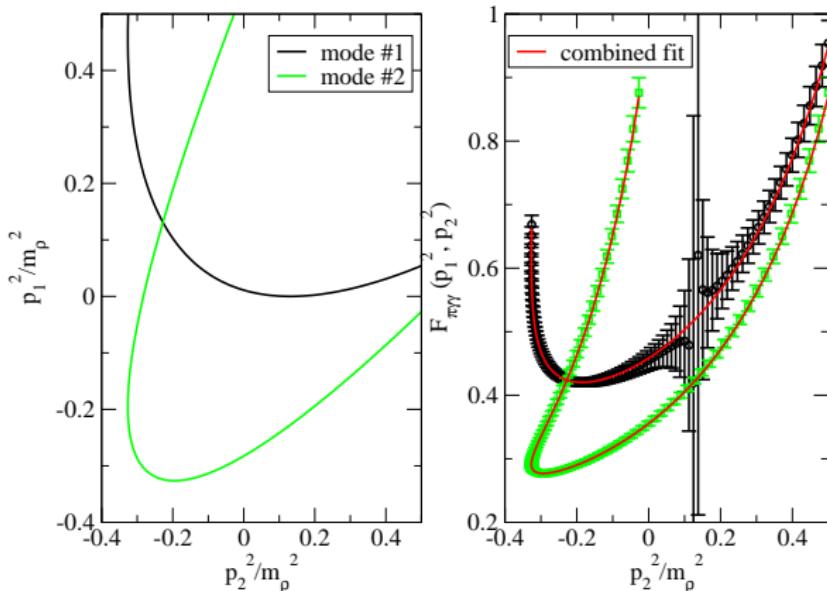
# Form factor

- $\mathcal{F}_{\pi^0\gamma\gamma} = \int dt e^{\omega t} \langle \Omega | T\{J_\mu(\vec{p}_1, t) J_\nu(\vec{p}_2, 0)\} | \pi^0(q) \rangle / \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta$
- tuning  $\omega$ :  $p_1^2 = \omega^2 - \vec{p}_1^2$     $p_2^2 = (E_\pi - \omega)^2 - \vec{p}_2^2$

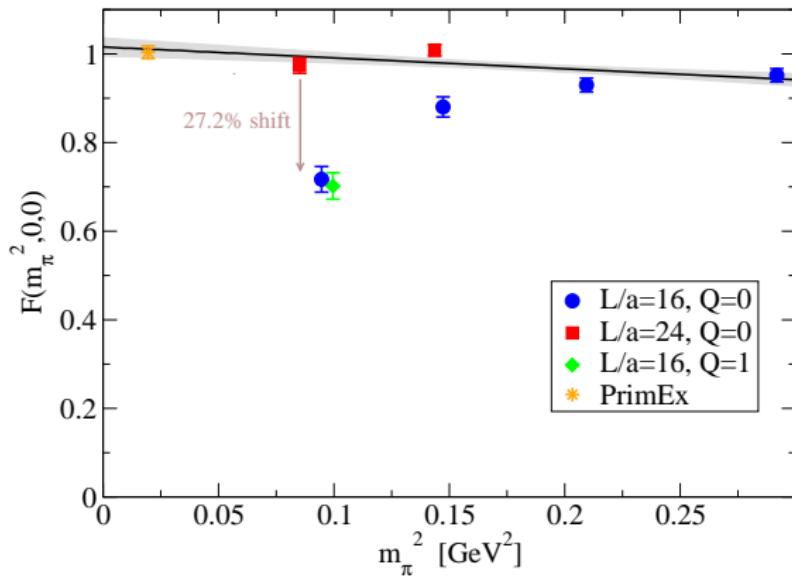


# Combined fit

- fit ansatz  $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2) + \sum_m c_m ((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)) + \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n$



# On-shell photon limit



- $F(m_{\pi}^2, 0, 0) \equiv \mathcal{F}_{\pi^0\gamma\gamma}(m_{\pi}^2, 0, 0)/\mathcal{F}_{\pi^0\gamma\gamma}^{\text{ABJ}}$
- data with  $m_{\pi}L \geq 4$ : consistent with ABJ and PrimEx
- $L = 16$ : smallest two quark mass,  $m_{\pi}L < 4$ , big FS effects
- FS effects checked at topological sector  $Q = 0$  and  $1$

# Expansion of matrix element

- expand the correlator into three hadronic matrix elements:

$$\langle J_\mu J_\nu \pi^0 \rangle \rightarrow \langle \Omega | J_\mu | V \rangle \langle V | J_\nu | \pi^0 \rangle \langle \pi^0 | \pi^0 | \Omega \rangle$$

- vector-meson electromagnetic coupling  $g_V$

$$\langle \Omega | J_\mu | V, p, \varepsilon \rangle = M_V F_V \varepsilon_\mu(p) = M_V^2 g_V \varepsilon_\mu(p)$$

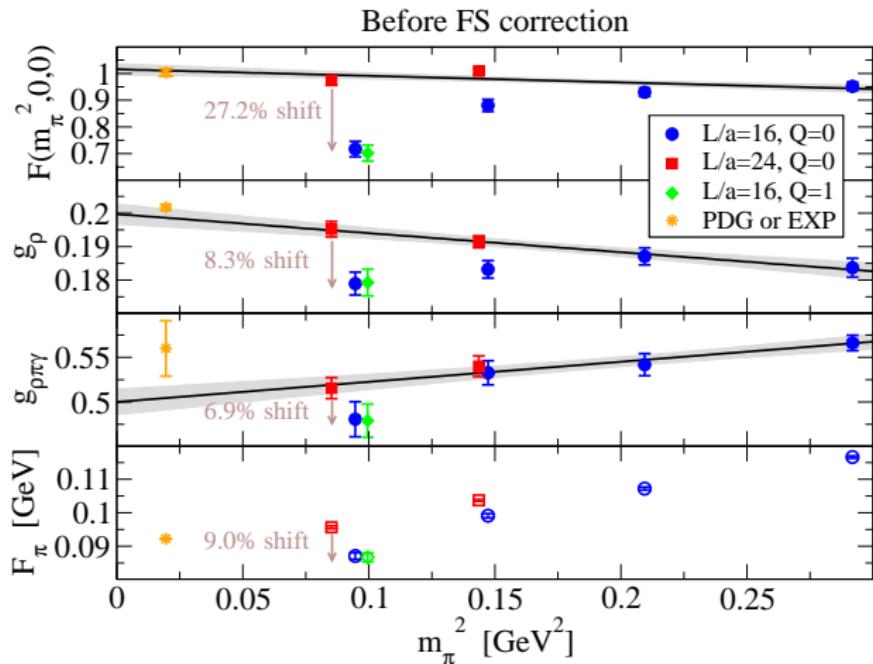
- $V\pi\gamma$  coupling  $g_{V\pi\gamma}$ ,  $V \rightarrow \pi\gamma$  decay

$$\langle V, \varepsilon_\mu(p_1) | j_\nu(0) | \pi^0(q) \rangle = \frac{g_{V\pi\gamma}}{M_V} \varepsilon_{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} K_V(p_2^2), \quad p_2 = q - p_1$$

- pion decay constant  $F_\pi$

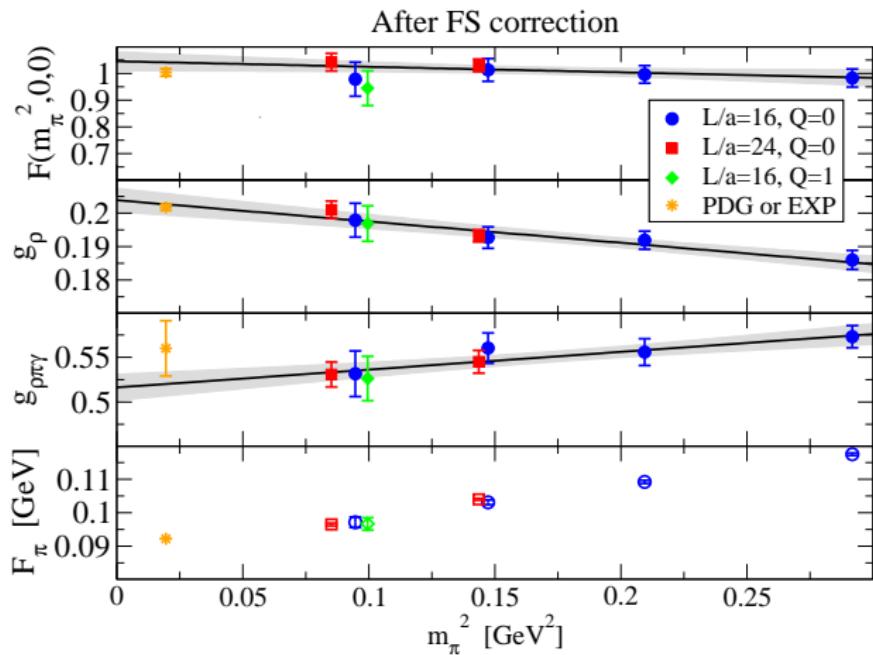
$$\langle \pi^0 | \pi^0 | \Omega \rangle = \frac{m_\pi^2}{2m_{u,d}} F_\pi$$

# Finite-size effects



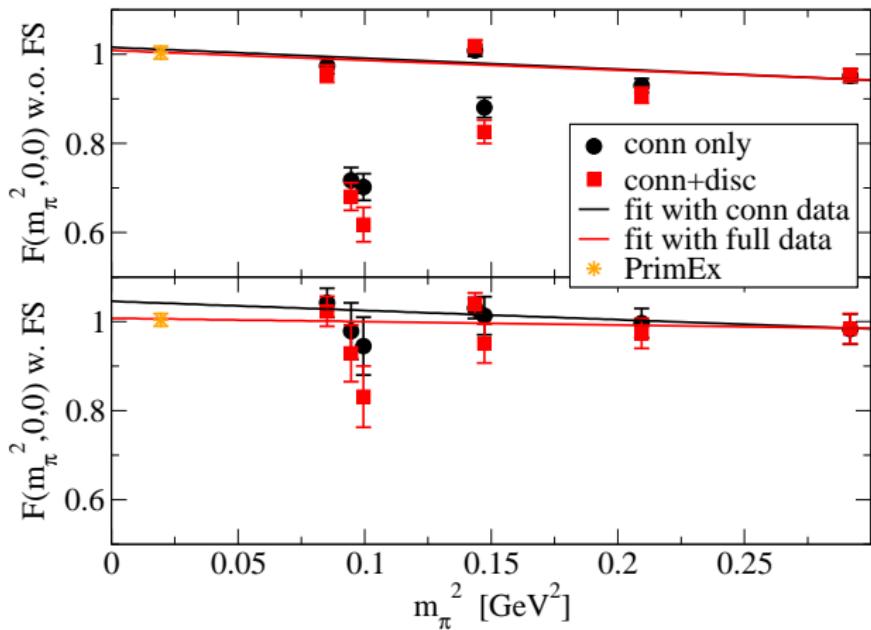
- FS effects accumulate and add up to a large effect

# Finite-size corrections



- FS corrections  $R_{\mathcal{O}} \equiv \mathcal{O}(\infty)/\mathcal{O}(L)$  with  $\mathcal{O} = g_\rho$ ,  $g_{\rho\pi\gamma}$  and  $F_\pi$
- assume that  $R_{F(m_\pi^2, 0, 0)} = R_{g_\rho} R_{g_{\rho\pi\gamma}} R_{F_\pi}$

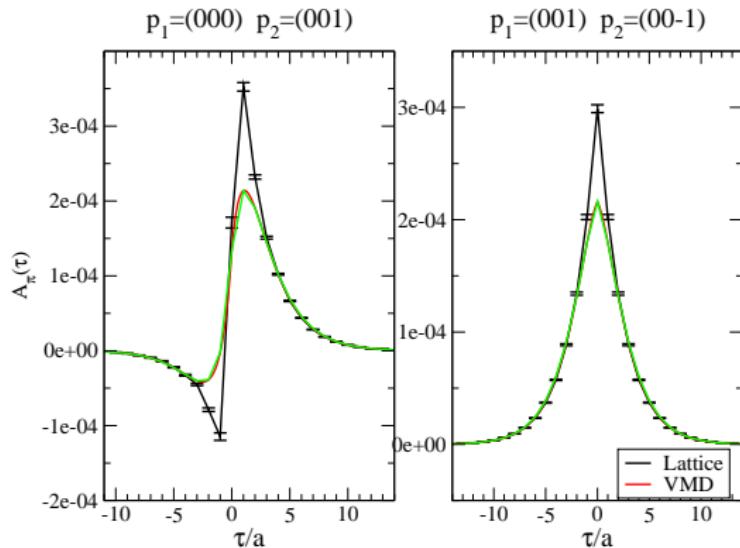
# Disconnected-diagram effects



- all-to-all propagator: control error of disc. contribution
- although not significant, conn+disc systematically shift down
- precision level (2% for form factor): disc. diagram should be included

# Lattice artifacts

- discrete data v.s. continuum case?
- right fig: turnover obtained; left fig: not obtained?



- disc. effects in VMD model: less than  $5 \times 10^{-4}$ , neglegiable
- turnover contributed by ground and excited states, not very large



# Conclusion

- after examining possible systematic effects

$$F(0, 0, 0) = 1.009(22)(29)$$

$$F(m_{\pi, \text{phy}}^2, 0, 0) = 1.005(20)(30)$$

$$\Gamma_{\pi^0 \gamma \gamma} = 7.93(29)(43) \text{ eV}$$

- ABJ anomaly and PrimEx measurement

$$F(0, 0, 0) = 1$$

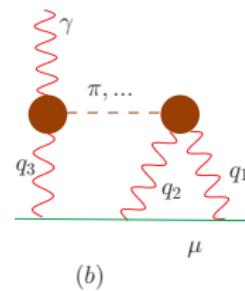
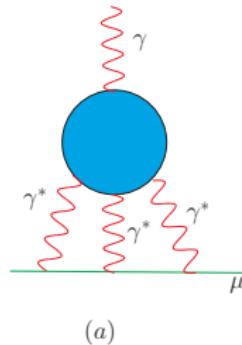
$$F(m_{\pi, \text{phy}}^2, 0, 0) = 1.004(14)$$

$$\Gamma_{\pi^0 \gamma \gamma} = 7.82(22) \text{ eV}$$

- $m_u \neq m_d$ , effects mixing of  $m_\pi$  with  $\eta$  and  $\eta'$  are not yet studied

## Motivated by muon $g_\mu - 2$

- Exp. determination of  $g_\mu - 2$  to 0.54 ppm [E821@BNL, PRD73, 2006]
- S.M. prediction of  $g_\mu - 2$  to 0.51 ppm [Jegerlehner, EPJC71, 2011]
- Discrepancy:  $3.3\sigma \Rightarrow$  New Physics ??
- HLbL is predicted to be dominant error in the next round



- Difficult: HLbL involves  $\langle J_\mu J_\nu J_\rho J_\sigma \rangle$
- Better to start with  $\pi^0(\eta, \eta') \rightarrow \gamma^* \gamma^*$

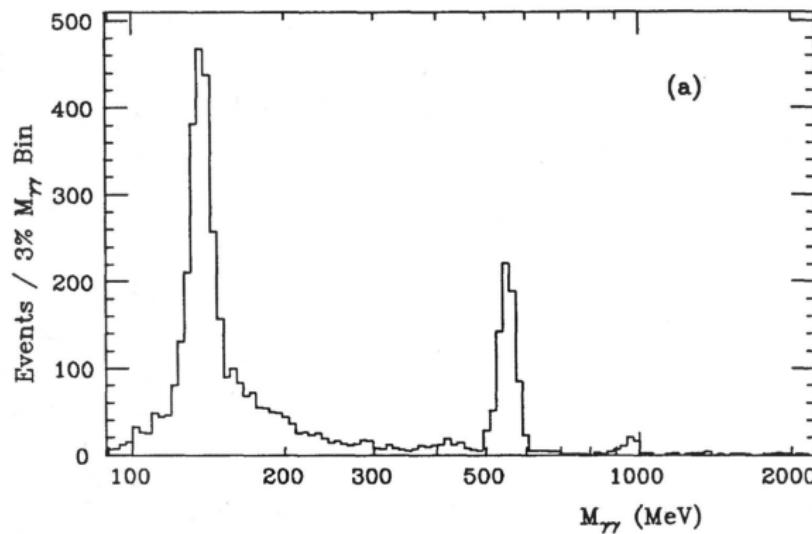
# $\pi^0$ contribution

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
$\pi^0, \eta, \eta'$	$85 \pm 13$	$82.7 \pm 6.4$	$83 \pm 12$	$114 \pm 10$	—	$114 \pm 13$	$99 \pm 16$
$\pi, K$ loops	$-19 \pm 13$	$-4.5 \pm 8.1$	—	—	—	$-19 \pm 19$	$-19 \pm 13$
$\pi, K$ loops + other subleading in $N_c$	—	—	—	$0 \pm 10$	—	—	—
axial vectors	$2.5 \pm 1.0$	$1.7 \pm 1.7$	—	$22 \pm 5$	—	$15 \pm 10$	$22 \pm 5$
scalars	$-6.8 \pm 2.0$	—	—	—	—	$-7 \pm 7$	$-7 \pm 2$
quark loops	$21 \pm 3$	$9.7 \pm 11.1$	—	—	—	2.3	$21 \pm 3$
total	$83 \pm 32$	$89.6 \pm 15.4$	$80 \pm 40$	$136 \pm 25$	$110 \pm 40$	$105 \pm 26$	$116 \pm 39$

- summary table [Jegerlehner, Nyffeler, Phys.Rept.477:1-110,2009]
  - $\pi^0(\eta, \eta') \rightarrow \gamma^* \gamma^*$  are consistent to total contributions
  - Among three PS mesons,  $\pi^0$  takes about 70%
  - calculation on the  $\pi^0 \rightarrow \gamma^* \gamma^*$  can be duplicated to the  $\eta, \eta'$  sector

# Non-perturbative nature

- Invariant mass spectrum for two-photon



- Three spikes presents three bound states:  $\pi^0$ ,  $\eta$ ,  $\eta'$
- Bound states  $\rightarrow$  confinement  $\rightarrow$  LQCD

# Rho mass

